

Ginsburg-Landau Theory of Solids and Supersolids

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Outline of the talk:

1. Introduction: the experiment, broken symmetries.....
2. Ginsburg-Landau theory of a supersolid
- 3a. SF to NS transition
- 3b. SF to SS transition and global phase diagram
4. Vacancies induced supersolid
5. Excitations in SS
6. Supersolid in other systems
7. Conclusions

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Moses Chan and **Tom Lubensky** for many helpful
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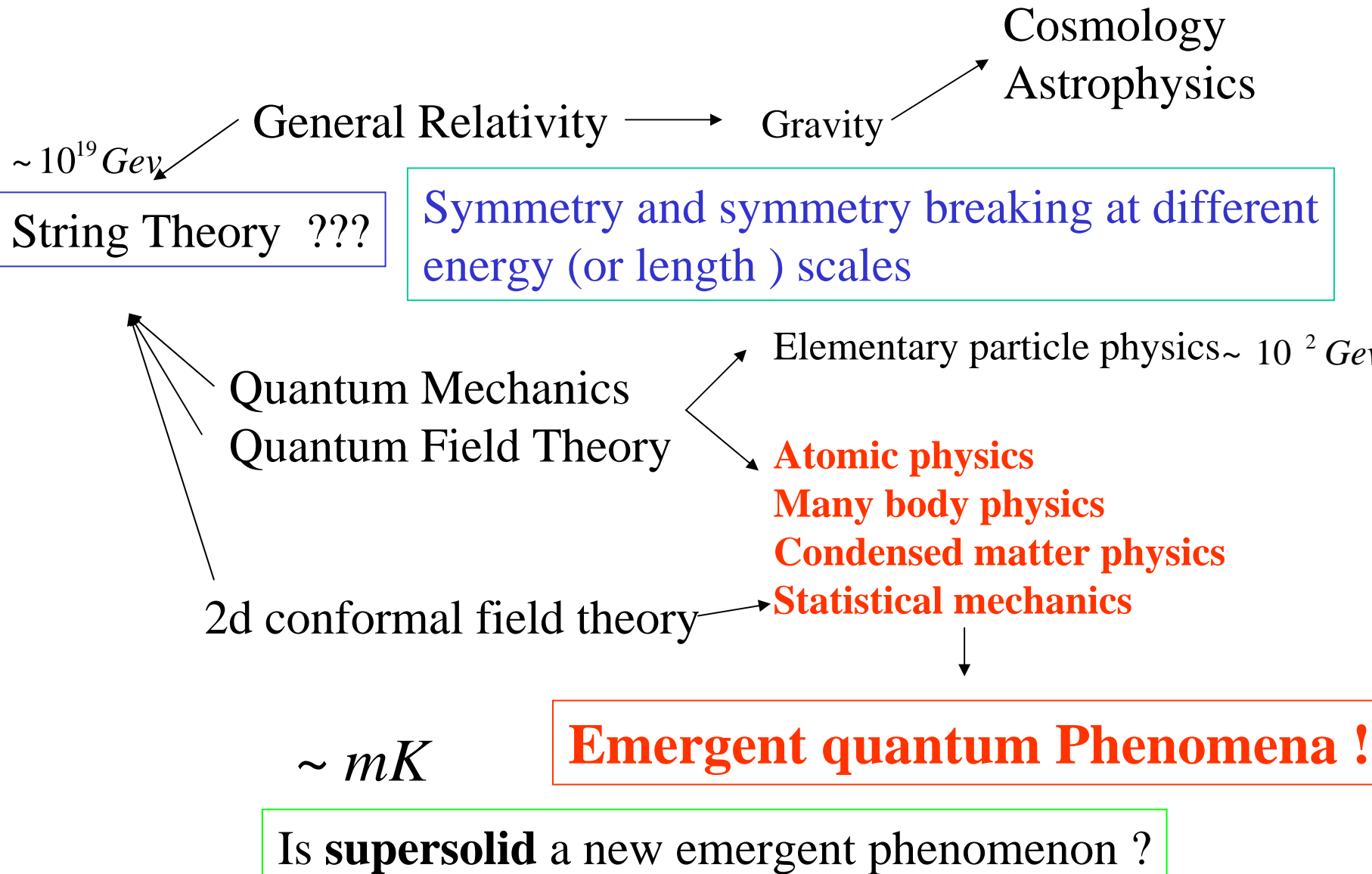
References:

Jinwu Ye, Phys. Rev. Lett. 97,125302, 2006

Jinwu Ye, cond-mat/0701694

Jinwu Ye, cond-mat/0603269,

1. Introduction



P.W. Anderson: More is different !
Emergent Phenomena !

Macroscopic quantum phenomena emerged in $\sim 10^{23}$ interacting atoms, electrons or spins

States of matter break different symmetries at low temperature

- Superconductivity
- Superfluid
- Quantum Hall effects
- Quantum Solids
- **Supersolid ?**
-???
- High temperature superconductors
- Mott insulators
- Quantum Anti-ferromagnets
- Spin density wave
- Charge density waves
- Valence bond solids
- Spin liquids ?
-???

Is the **supersolid** a new state of matter ?

What is a liquid ?

A liquid can flow with some viscosity

It breaks **no** symmetry, it exists only at **high** temperatures

At **low** temperatures, any matter **has to** have some orders which break some kinds of symmetries.

What is a solid ?

A solid can **not** flow \vec{G} : Reciprocal lattice vectors

Density operator: $n(\vec{x}) = n_0 + \sum_{\vec{G}} n_{\vec{G}} e^{i\vec{G} \cdot \vec{x}}$

Breaking translational symmetry: \Rightarrow Lattice phonons \vec{u}

H_2O, H_2, \dots

Essentially all substances take solids **except** ${}^4\text{He}$, ${}^3\text{He}$

What is a superfluid ?

A superfluid can **flow** without **viscosity**

Complex order parameter: $\psi = |\psi| e^{i\theta}$

Breaking Global U(1) symmetry: \Rightarrow Superfluid phonons θ

What is a **supersolid** ?

A **supersolid** is a new state which has both crystalline order and superfluid order.

Can a supersolid exist in nature, especially in He4 ?

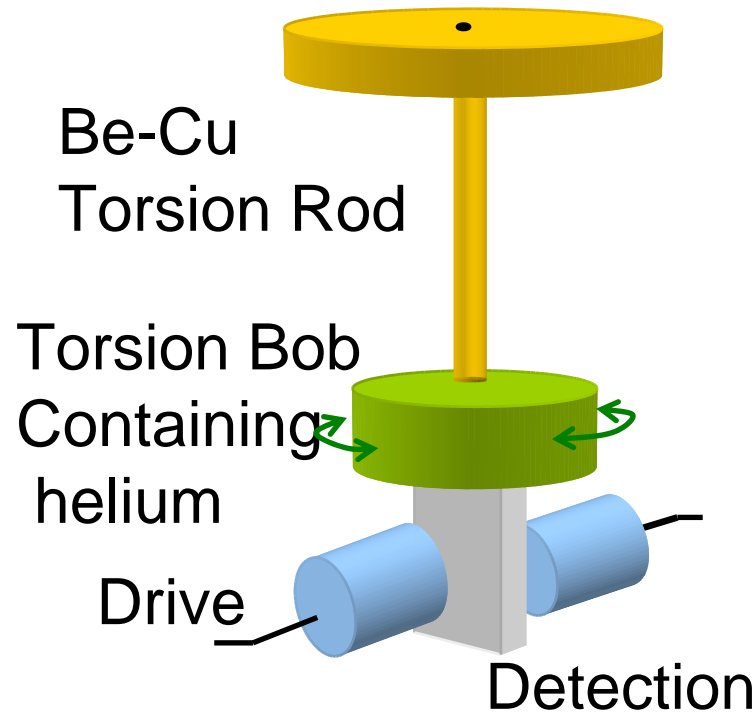
Large quantum fluctuations in He4 make it possible

The **supersolid** state was theoretically speculated in 1970:

- Andreev and I. Lifshitz, 1969.* Bose-Einstein Condensation (**BEC**) of **vacancies** leads to supersolid, classical hydrodynamics of vacancies.
- G. V. Chester, 1970,* Wavefunction with both **BEC** and crystalline order, a supersolid cannot exist without vacancies or interstitials
- A. J. Leggett, 1970,* **Non-Classical Rotational Inertial (NCRI)** of supersolid He4 $I = I_{cl}(1 - \rho_s / \rho)$, $\rho_s / \rho \leq 10^{-4}$
- quantum **exchange process** of He atoms can also lead to a supersolid even in the **absence** of vacancies,
- W. M. Saslow, 1976,* improve the upper bound $\rho_s / \rho \leq 10^{-2}$

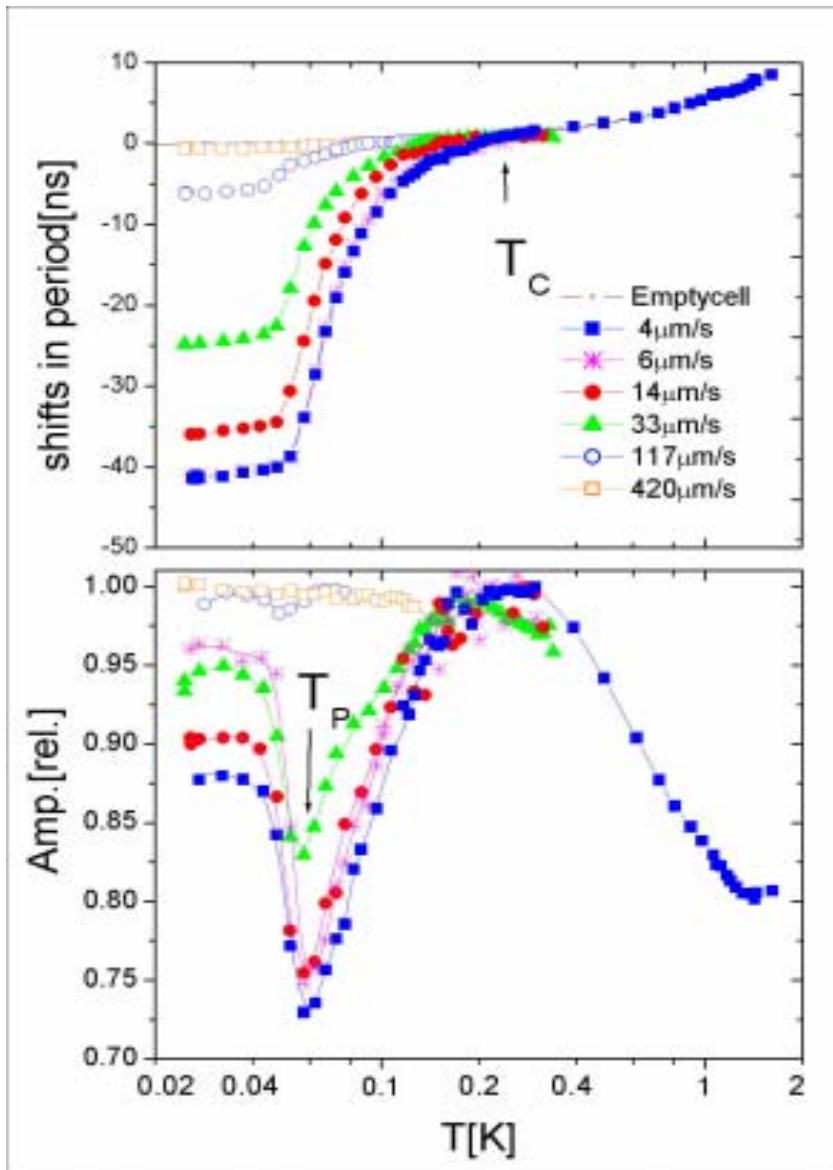
Over the last 35 years, a number of experiments have been designed to search for the supersolid state **without success**.

Torsional oscillator is ideal for the detection of superfluidity



$$\tau_o = 2\pi \sqrt{\frac{I}{K}}$$

$$I = I_{cl} (1 - \rho_s / \rho)$$



Science 305, 1941 (2004)

Solid He4 at 51 bars

1 ~ 2% NCRI appears below
0.25K

Strong $|v|_{max}$ dependence
(above $14 \mu\text{m/s}$)

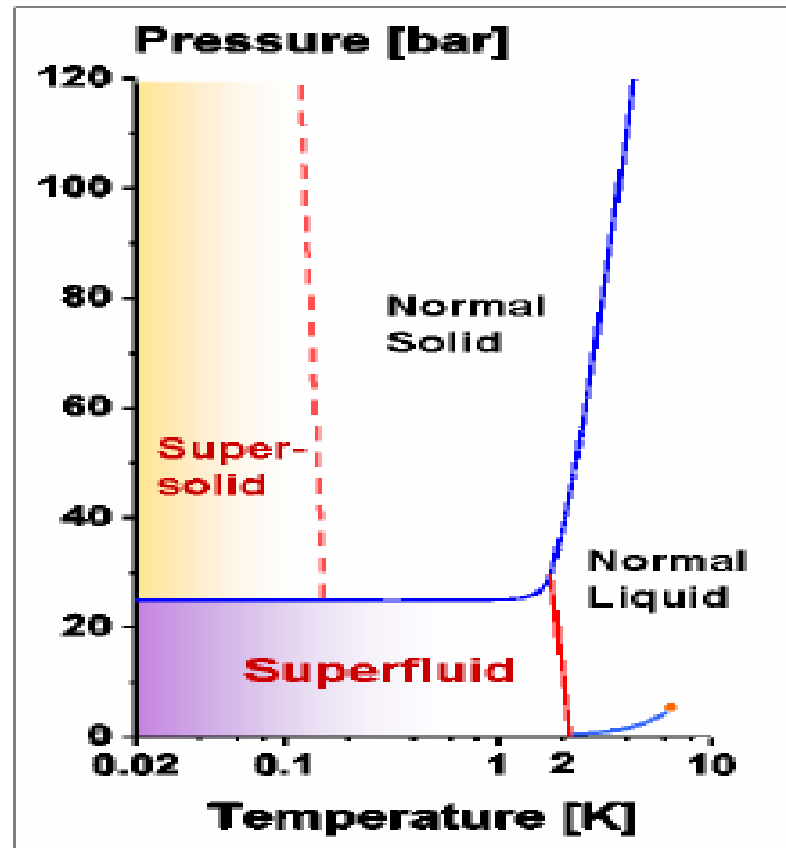
$$\tau = 2\pi \sqrt{\frac{I}{K}}$$

$$I = I_{cl} (1 - \rho_s / \rho)$$

Very recently, there are three **experimental** groups one in US, two in Japan
Confirmed (?) PSU's experiments.

1. *A.S. Rittner and J. D. Reppy, cond-mat/0604568*
2. *M. Kubota et al*
3. *K. Shirahama et al*

Possible phase diagram
of Helium 4



PSU's experiments have rekindled great **theoretical** interests in the possible supersolid phase of He4

(1) **Numerical** (Path Integral quantum Monte-Carlo) approach:

- D. M. Ceperley, B. Bernu, Phys. Rev. Lett. 93, 155303 (2004);
- N. Prokof'ev, B. Svistunov, Phys. Rev. Lett. 94, 155302 (2005);
- E. Burovski, E. Kozik, A. Kuklov, N. Prokof'ev, B. Svistunov, Phys. Rev. Lett., 94,165301(2005).
- M. Boninsegni, A. B. Kuklov, L. Pollet, N. V. Prokof'ev, B. V. Svistunov, and M. Troyer, Phys. Rev. Lett. {\bf 97}, 080401 (2006).

**Superfluid flowing
in grain boundary ?**

(2) **Phenomenological** approach:

- Xi Dai, Michael Ma, Fu-Chun Zhang, Phys. Rev. B 72, 132504 (2005)
- P. W. Anderson, W. F. Brinkman, David A. Huse, Phys. Rev. Lett. 96, 055301 (2006)
- A. T. Dorsey, P. M. Goldbart, J. Toner, Science 18 Nov. 2005
-

Supersolid ?

•••••

Uncertainty Relation: $[x, p] = i, \Delta x \bullet \Delta p \geq 1$

Cannot measure position and momentum precisely simultaneously !

$$[N_b, \phi] = i, \quad \Delta N_b \bullet \Delta \phi \geq 1$$

Cannot measure phase and density precisely simultaneously !
 $n(\vec{x})$

Superfluid: **phase order** in ϕ

Solid: **density order** in $n(\vec{x})$

How to reconcile the **two extremes** into a supersolid ?

The solid is **not** perfect: has either **vacancies** or **interstitials** whose BEC may lead to a supersolid

2. Ginsburg-Landau (GL) Theory of a supersolid

Construct GL theory:

Expanding free energy in terms of **order parameters** near critical point consistent with **the symmetries** of the system

(1) GL theory of Liquid to superfluid transition:

Complex order parameter:

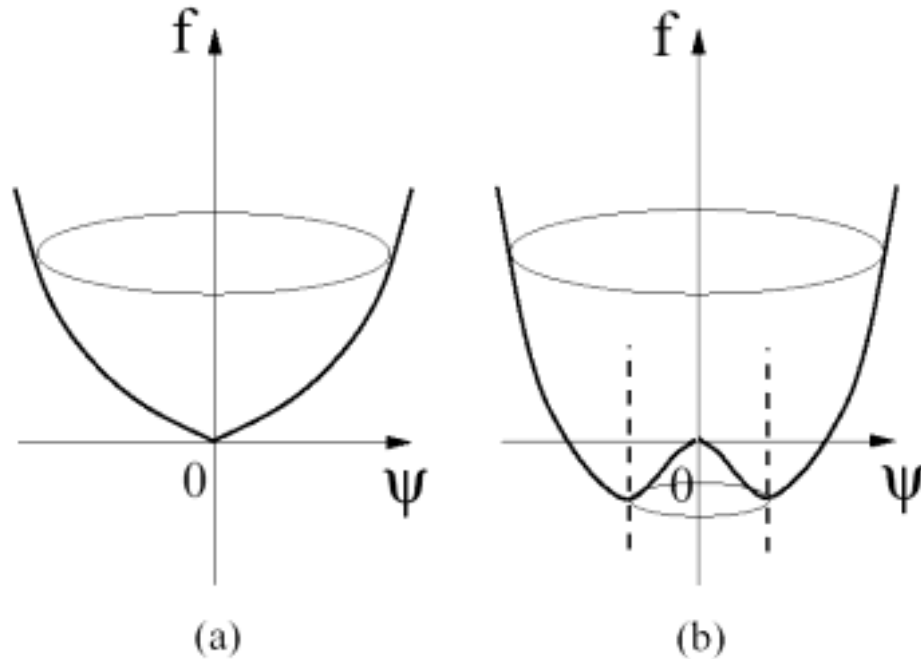
$$\psi = |\psi| e^{i\theta}$$

~~$$|\psi|^3$$~~

$$f_{L-SF} = K|\nabla\psi|^2 + t|\psi|^2 + u|\psi|^4 + \dots$$

$$t = T - T_c$$

Invariant under the U(1) symmetry: $\theta \rightarrow \theta + \phi$



(a) In liquid:

$$t > 0, \langle \psi \rangle = 0$$

U(1) symmetry is respected

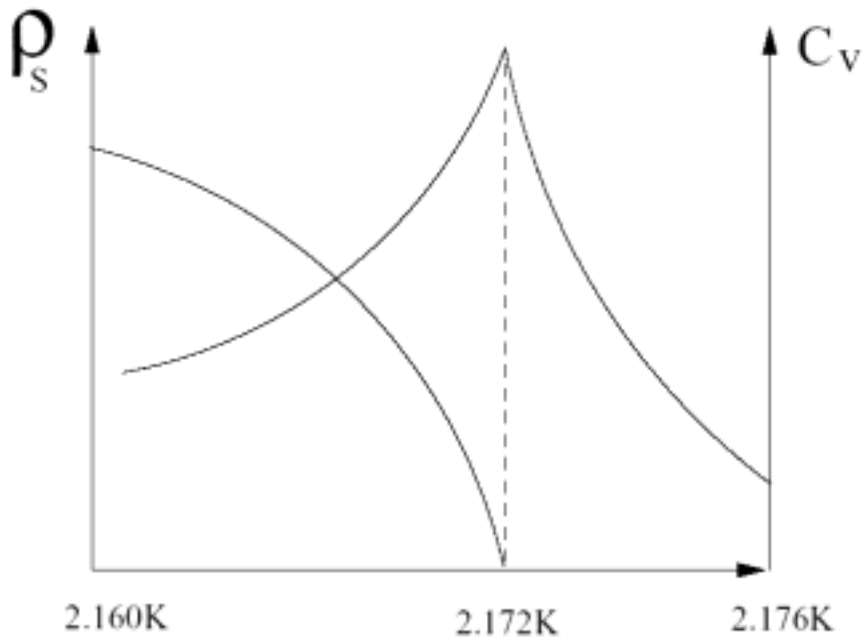
(b) In superfluid:

$$t < 0, \langle \psi \rangle \neq 0$$

U(1) symmetry is broken

Both sides have the translational symmetry: $\psi(\vec{x}) \rightarrow \psi(\vec{x} + \vec{a})$

3d XY model to describe the λ (cusp) transition



Greywall and Ahlers, 1973
Ahler, 1971

$$C_v \sim |t|^{-\alpha}, \rho_s \sim |t|^\nu$$

$$\alpha = -0.013 < 0, \nu = 0.672$$

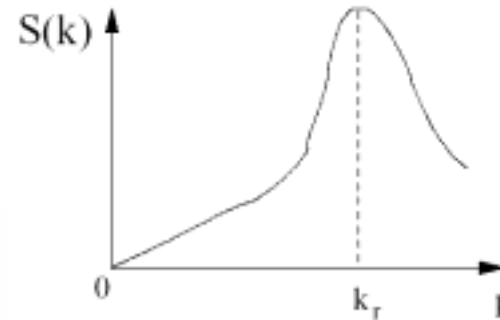
(2) GL theory of Liquid to solid transition:

Density operator: $n(\vec{x}) - n_0 = \delta n(\vec{x}) = \sum_{\vec{G}} n_{\vec{G}} e^{i\vec{G} \cdot \vec{x}}$

Order parameter: $n_{\vec{G}}, \vec{G} \neq 0$

$$f_{L-NS} = \sum_{\vec{G}} \frac{1}{2} r_{\vec{G}} |n_{\vec{G}}|^2 - w \sum_{\vec{G}_1, \vec{G}_2, \vec{G}_3} n_{\vec{G}_1} n_{\vec{G}_2} n_{\vec{G}_3} \delta_{\vec{G}_1 + \vec{G}_2 + \vec{G}_3, 0}$$

$$+ u \sum_{\vec{G}_1, \vec{G}_2, \vec{G}_3, \vec{G}_4} n_{\vec{G}_1} n_{\vec{G}_2} n_{\vec{G}_3} n_{\vec{G}_4} \delta_{\vec{G}_1 + \vec{G}_2 + \vec{G}_3 + \vec{G}_4, 0} + \dots$$



$$r_{\vec{G}} = r + c(G^2 - k_r^2)^2, \quad S(k) \sim 1/r_{\vec{G}}$$

$$\delta n(\vec{x}) \not\rightarrow -\delta n(\vec{x})$$

Invariant under translational symmetry:

$$n(\vec{x}) \rightarrow n(\vec{x} + \vec{a}), \quad n_{\vec{G}} \rightarrow n_{\vec{G}} e^{i\vec{G} \cdot \vec{a}} \quad \vec{a} \text{ is any vector}$$

In liquid: $r > 0, \langle n_{\vec{G}} \rangle = 0$

In solid: $r < 0, \langle n_{\vec{G}} \rangle \neq 0$

Translational symmetry is **broken** down to the lattice symmetry:

$$\vec{a} = \vec{R}, \quad \vec{G} \cdot \vec{R} = 2\pi \quad \vec{R} \quad \text{is any lattice vector}$$

$$n(\vec{x}) \rightarrow n(\vec{x} + \vec{R}), \quad n_{\vec{G}} \rightarrow n_{\vec{G}} e^{i\vec{G} \cdot \vec{R}} = n_{\vec{G}}$$

NL to **SF** transition

Complex OP

at $k = 0$

even order terms

2nd order transition

NL to **NS** transition

Density OP $n_{\vec{G}}, \vec{G} \neq 0$

at reciprocal lattice vector \vec{G}

cubic term,

first order

(3) GL theory of a supersolid:

Density operator:
$$\delta n(\vec{x}) = \sum_{\vec{G}} n_{\vec{G}} e^{i\vec{G}\cdot\vec{x}}$$

Complex order parameter: $\Psi(\mathbf{x})$: Vacancy or interstitial

$$f = f_{L-SF} + f_{L-NS} + f_{\text{int}}$$

$$f_{\text{int}} = g \delta n(x) |\psi(x)|^2 + v (\delta n(x))^2 |\psi(x)|^2 \dots$$

Invariant under: $n(\vec{x}) \rightarrow n(\vec{x} + \vec{a}), \psi(\vec{x}) \rightarrow \psi(\vec{x} + \vec{a}),$
 $\psi \rightarrow \psi e^{i\phi}$

Under $\delta n(x) \rightarrow -\delta n(x), g \rightarrow -g, v \rightarrow v$

Two competing orders \implies Repulsive: $v > 0$

$g > 0$: Vacancy case

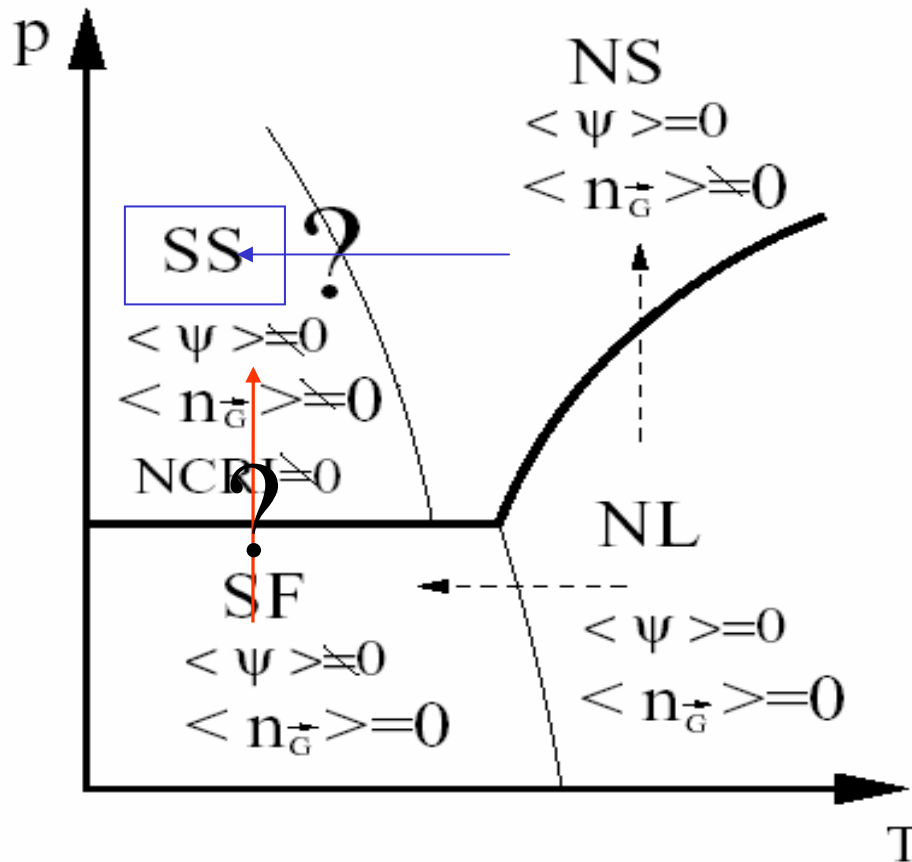
$g < 0$: Interstitial case

In supersolid:

$$\langle n_{\vec{G}} \rangle \neq 0, \langle \psi \rangle \neq 0$$

In the NL, $t > 0$, ψ has a gap, can be **integrated out**.

In the NL, $\langle n(x) \rangle = n_0$, $\delta n(x)$ has a gap, can be **integrated out**



3. The SF to NS or SS transition

In the SF state, $t < 0, \Delta_r > 0$

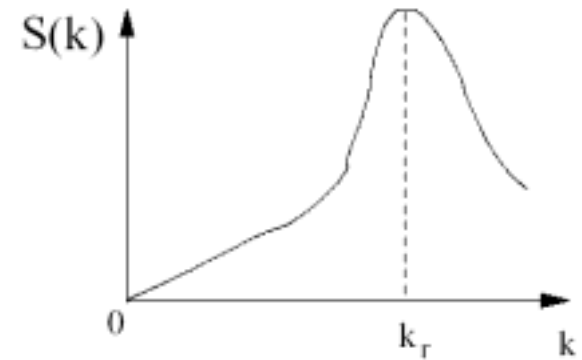
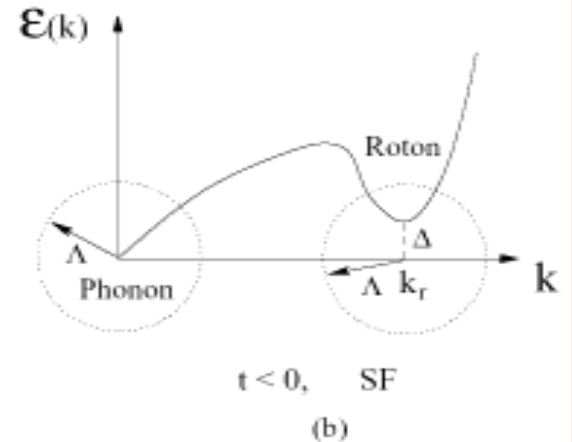
BEC: $\langle \psi_1 \rangle = a \neq 0$ breaks U(1) symmetry

$$k_r \sim \frac{2\pi}{a} \sim \frac{2\pi}{3.17 \text{ \AA}}$$

As $P \uparrow$, the roton minimum gets deeper and deeper, $\Delta_r \downarrow$ as detected in neutron scattering experiments

Feynmann relation:

$$\omega(q) \sim \frac{q^2}{2mS_n(q)}$$



The first maximum peak in $S_n(q) \Rightarrow$ the roton minimum in $\omega(q)$

Two possibilities:

$$(1) \quad \langle \psi \rangle = 0$$

Commensurate Solid,

SF to NS transition

No SS

$$(2) \quad \langle \psi \rangle \neq 0$$

In-Commensurate Solid with either **vacancies** or **interstitials**

There is a SS !

SF to SS transition

Which scenario will happen *depends on* the sign and strength of the coupling g , will be analyzed further in a few minutes

Let's focus on **case (1) first:**

I will **explicitly** construct a Quantum Ginzburg-Landau (QGL) action to describe **SF to NS transition**

3a. The SF to NS transition

Inside the SF: $\mathcal{L}[\delta n, \theta] = i\delta n \partial_\tau \theta + \frac{1}{2} \rho_s (\nabla \theta)^2 + \frac{1}{2} \delta n V_n(\vec{q}) \delta n$

Where:

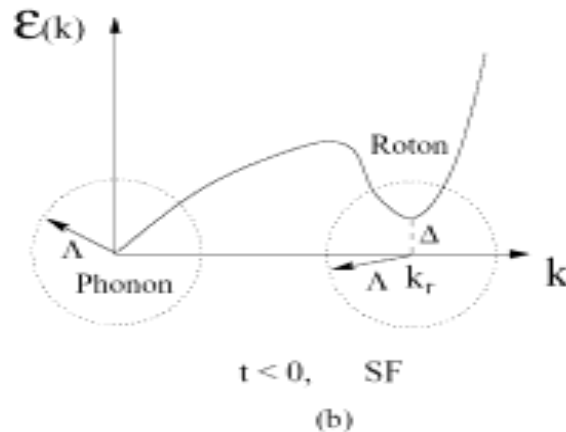
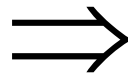
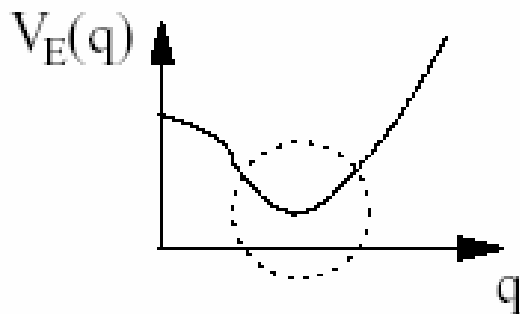
$$V_n(q) = a - bq^2 + cq^4$$

Phase representation:

$$\mathcal{L}[\theta] = \frac{1}{2V_n(\vec{q})} (\partial_\tau \theta)^2 + \frac{1}{2} \rho_s (\nabla \theta)^2$$

Dispersion Relation:

$$\omega^2 = [\rho_s V_n(\vec{q})] q^2 = \rho_s q^2 (a - bq^2 + cq^4)$$

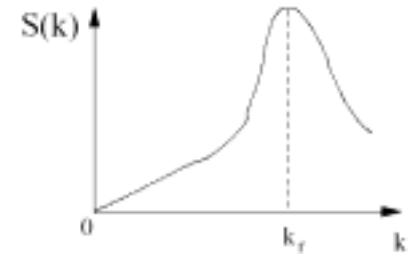


Neglecting vortex excitations:

Density representation: $\mathcal{L}[\delta n] = \frac{1}{2} \delta n(-\vec{q}, -\omega_n) \left[\frac{\omega_n^2}{\rho_s q^2} + V_n(\vec{q}) \right] \delta n(\vec{q}, \omega_n)$

Feymann Relation:

$$\omega(q) = \frac{\rho_s \pi q^2}{2S_n(q)}$$



Structure function:

$$S_n(q) = \rho_s q \pi / 2v(q)$$

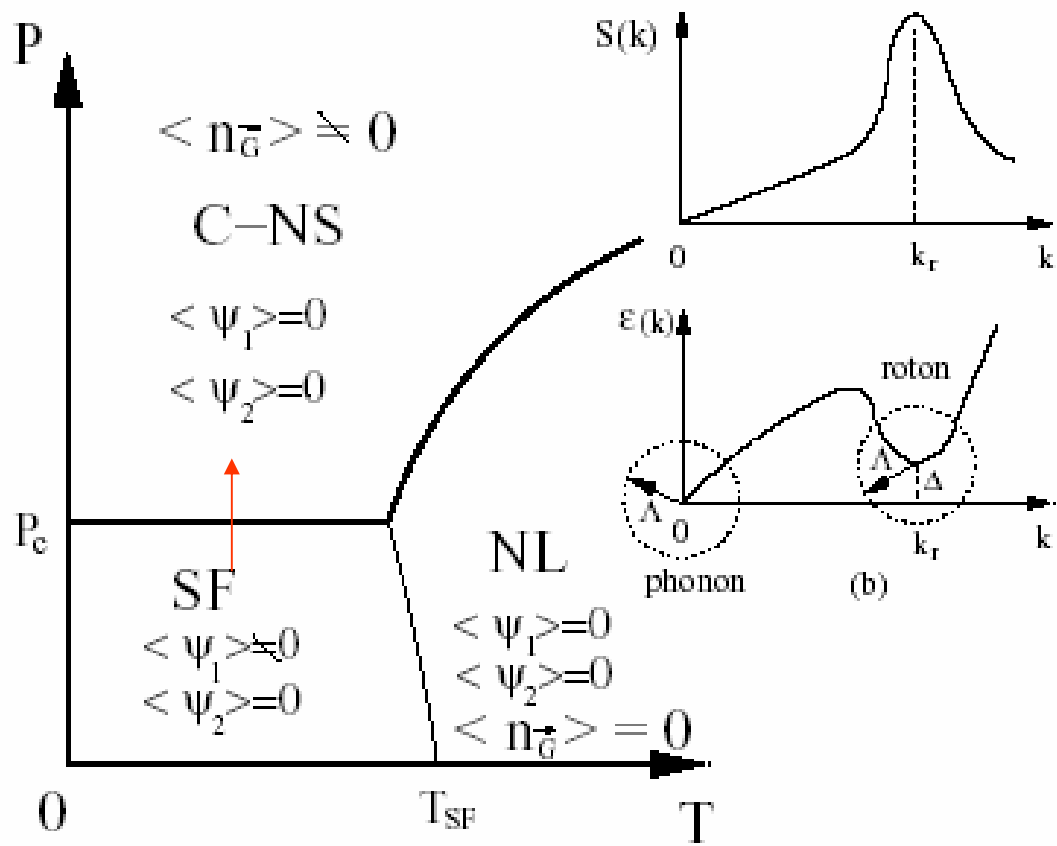
Including vortex excitations: QGL to describe SF to NS transition:

$$\mathcal{L}[\delta n] = \frac{1}{2} \delta n [A_n \omega_n^2 + r + c(q^2 - q_0^2)^2] \delta n - w(\delta n)^3 + u(\delta n)^4 + \dots$$

$$r \sim p_{c1} - p \quad A_\rho \sim \frac{1}{\rho_s q_0^2}$$

SF: $r > 0, \langle \psi \rangle \neq 0, \langle \delta n \rangle = 0$

NS: $r < 0, \langle \psi \rangle = 0, \langle \delta n \rangle = \sum_{\vec{G}'} n_{\vec{G}'} e^{i\vec{G}' \cdot \vec{x}}$



(a)

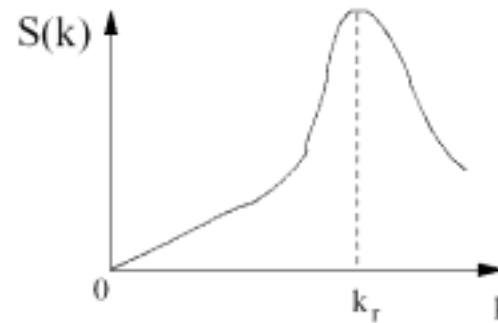
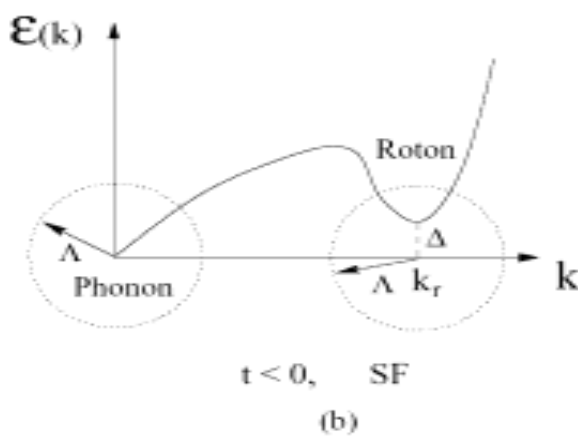
(b)

3b. The SF to SS transition

(2) $\langle \psi \rangle \neq 0$ There is a SS !

SF to SS transition

ψ : Vacancies or Interstitials !



Feymann relation:

$$\omega(q) \sim \frac{q^2}{2mS_n(q)}$$

The first maximum peak in $S_n(q) \Rightarrow$ the roton minimum in $\omega(q)$

The n lattice and ψ Superfluid density wave (SDW) formations happen **simultaneously**

$$\Delta_r > 0, \langle \psi_1 \rangle \neq 0, \langle \psi_2 \rangle = 0, \langle n_{\vec{G}} \rangle = 0$$

SF phase

$$\Delta_r < 0, \langle \psi_1 \rangle \neq 0, \langle \psi_2 \rangle \neq 0, \langle n_{\vec{G}} \rangle \neq 0$$

SS phase

$$n(\vec{x}) = n_0 + \sum_{\vec{G}} n_{\vec{G}} e^{i\vec{G} \cdot \vec{x}}$$

$$G = k_n = k_r$$

$$k_r \sim \frac{2\pi}{a} \sim \frac{2\pi}{3.17 \text{ \AA}}$$

$$\psi(\vec{x}) = \psi_1 + \sum_{\vec{G}} \psi_{\vec{G}} e^{i\vec{G} \cdot \vec{x}}, \quad \psi_2 = \sum_{\vec{G}} \psi_{\vec{G}} e^{i\vec{G} \cdot \vec{x}}$$

SS is still **invariant** under translations by lattice vectors: \vec{R}

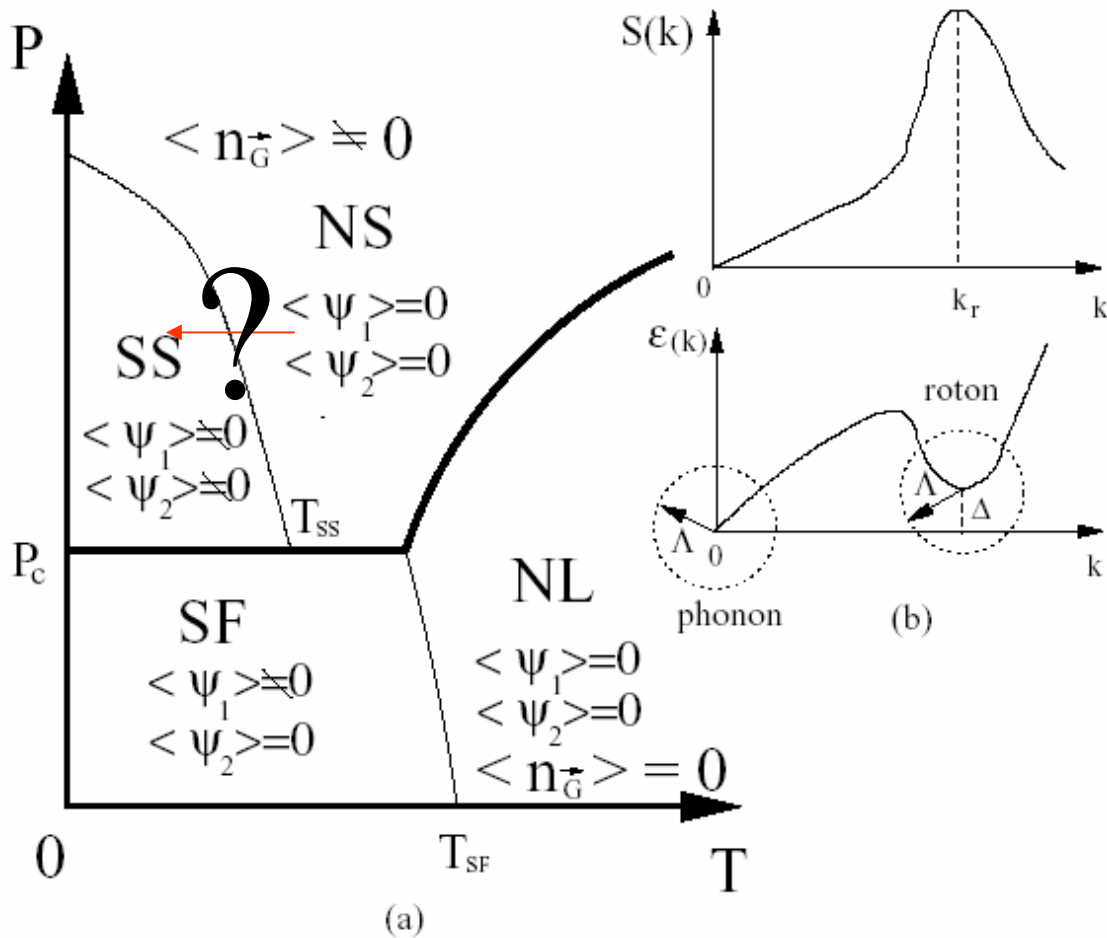
$$n(\vec{x}) \rightarrow n(\vec{x} + \vec{R}), \quad n_{\vec{G}} \rightarrow n_{\vec{G}} e^{i\vec{G} \cdot \vec{R}} = n_{\vec{G}}$$

$$\psi(\vec{x}) \rightarrow \psi(\vec{x} + \vec{R}), \quad \psi_{\vec{G}} \rightarrow \psi_{\vec{G}} e^{i\vec{G} \cdot \vec{R}} = \psi_{\vec{G}}$$

Superfluid Density Wave : $\rho = |\psi|^2 = |\psi_1 + \psi_2|^2$

Vacancy SDW or Interstitial SDW

Global phase diagram of He4 in case (2)



$$\omega_q = \frac{q^2}{2mS_q}$$

The phase diagram will be confirmed from the **NS** side

4. Vacancy induced supersolid and NS to SS transition

Looking the effects of g and ν

$g > 0$, solid is the **vacancy** type, NS-v, $\epsilon_v < \epsilon_i$

$g < 0$, solid is the **interstitial** type, NS-i, $\epsilon_v > \epsilon_i$

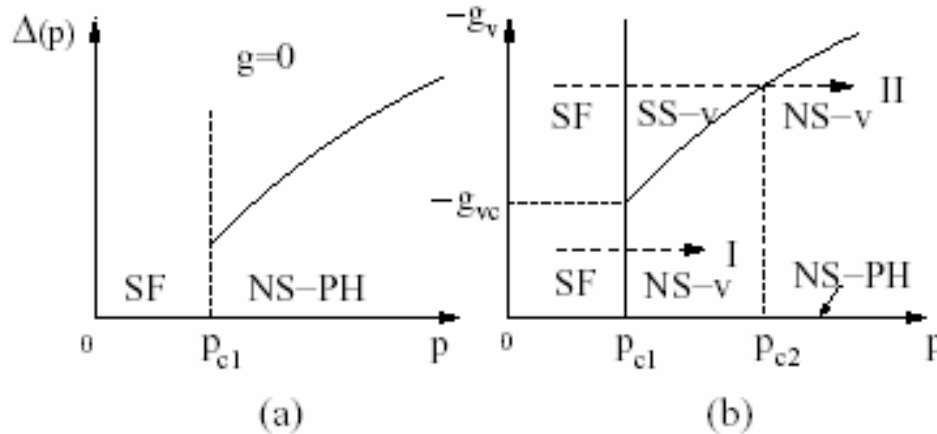
$g \rightarrow 0$, solid has the P-H symmetry, NS-PH

$$\nu > 0 \Rightarrow t \rightarrow t + \nu(p) \sum_{\vec{G}} |n(\vec{G})|^2$$

$$t \sim T - T_{XY}$$

$$t = T + \Delta(p), \Delta(p) = \nu(p) \sum_{\vec{G}} |n(\vec{G})|^2 - T_{XY} > 0$$

We assume the gap $\Delta(\mathbf{p})$ is the increasing function of p



Taking the NS-PH as **the reference state**, tuning on g

A perturbative estimate on $\epsilon_\mu(0)$ at the origin $\vec{k} = 0$:

$$\epsilon_\mu(0) = t - g_\mu^2 P \frac{n^2(\vec{G})}{K_\mu G^2} + g_\mu^3 \sum'_{\vec{G}_1} \sum'_{\vec{G}_2} \frac{n(\vec{G}_1)n(\vec{G}_2)n(-\vec{G}_1 - \vec{G}_2)}{K_\mu G_1^2 K_\mu G_2^2} + \dots (4)$$

$$n(\vec{G}) = e^{-G^2 a^2 / 4\alpha}$$

$$\mu = v, i$$

For **vacancies**, $g_v < 0$

$$\epsilon_v(0) = t - g_v^2 - |g_v|^3 - |g_v|^4 + \dots = t - f_v(g_v)$$

$f_v(g_v) > f_v(0) = 0$ is an **increasing** function of g_v

For **interstitials**, $g_i > 0$

$$\epsilon_i(0) = t - g_i^2 + g_i^3 - g_i^4 + \dots = t - f_i(g_i)$$

It is hard to judge the behavior of $f_i(g_i)$ except $f_i(0) = 0$

Where:

$$t = T + \Delta(p), \Delta(p) = v(p) \sum_{\vec{G}} |n(\vec{G})|^2 - T_{XY} > 0$$

Vacancies induced supersolids:

$$t_{\psi_v} = T + \Delta(p) - f_v(g_v) = T - T_{SS-v}$$

$$T_{SS-v} = f_v(g_v) - \Delta(p)$$

Defining a critical value $g_{vc} : f_v(g_{vc}) = \Delta(p_{c1})$

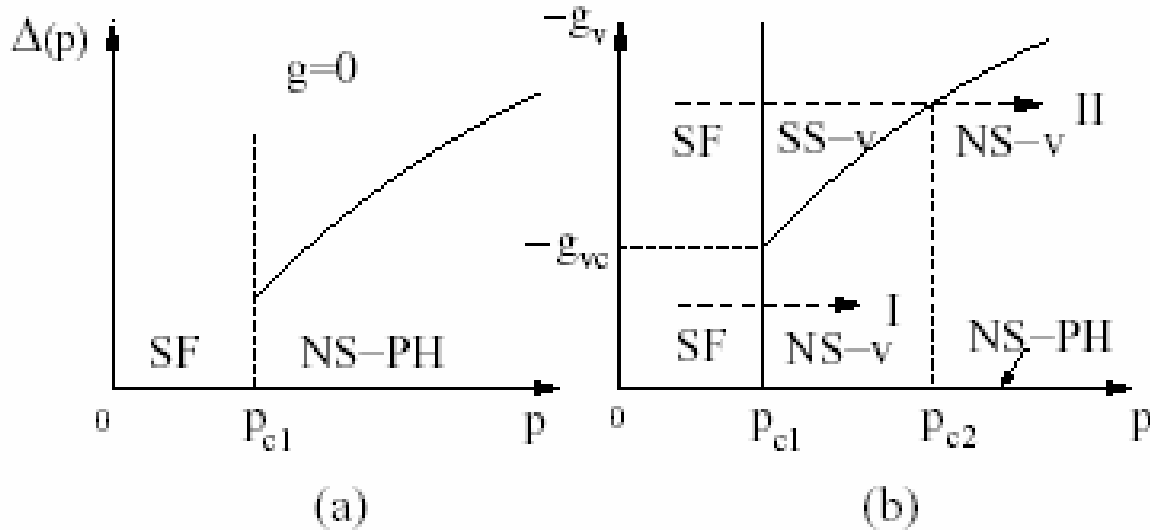
(a) $|g_v| < |g_{vc}|, T_{SS-v} < 0, \langle \psi_v \rangle = 0, NS - v$

(b) $|g_v| > |g_{vc}|, T_{SS-v}(p_{c1}) = f_v(g_v) - \Delta(p_{c1}) > 0$

$p \uparrow \quad T_{SS-v}(p_{c2}) = f_v(g_v) - \Delta(p_{c2}) = 0$

$$T_{SS-v}(p) = f_v(g_v) - \Delta(p) = \Delta(p_{c2}) - \Delta(p)$$

is an **effective measure** of the gap $\Delta(p)$



Setting $\langle \psi_1 \rangle = a e^{i\theta_1}$, $\langle \psi_2 \rangle = e^{i\theta_2} \sum_{m=1}^P \psi_m e^{i\vec{Q}_m \cdot \vec{x}}$, $Q_m \sim G$ into the GL,

We study the effects of \mathbf{n} lattice on $\psi = \psi_1 + \psi_2$

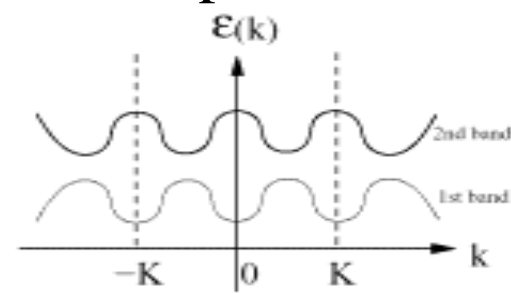
$$f_{\text{int}} = g \delta n(x) |\psi(x)|^2 + \dots$$

The lowest energy **ground state** must satisfy:

- (1) Ψ has to be real, \vec{Q}_m has to be paired as **anti-nodal** points
- (2) $\vec{Q}_m, m = 1, \dots, P$ are the P **shortest** reciprocal lattice vectors

Bloch theorem:

$$\boxed{\varepsilon(\vec{K} = 0) = \varepsilon(\vec{K} = \vec{Q}_m)}$$



Ψ_1 and Ψ have to condense at the same time

- (3) Point group symmetry: $\Delta_m = \Delta = \text{real}$
- (4) The attractive $g_v < 0$ **favors** $\psi(x = \vec{R}/2) \sim 0$

\Rightarrow

$$\psi_{ss-v} = \psi_0 \left(1 + \frac{2}{P} \sum_{m=1}^{P/2} \cos \vec{Q}_m \cdot \vec{x} \right)$$

$$\psi_0 = ae^{i\theta}$$

Similarly, Interstitials induced supersolid SS-i:

$$\psi_{ss-i} = \psi_0 \left(1 - \frac{2}{P} \sum_{m=1}^{P/2} \cos \vec{Q}_m \cdot \vec{x} \right)$$

For both SS-v (+) or SS-i (-):

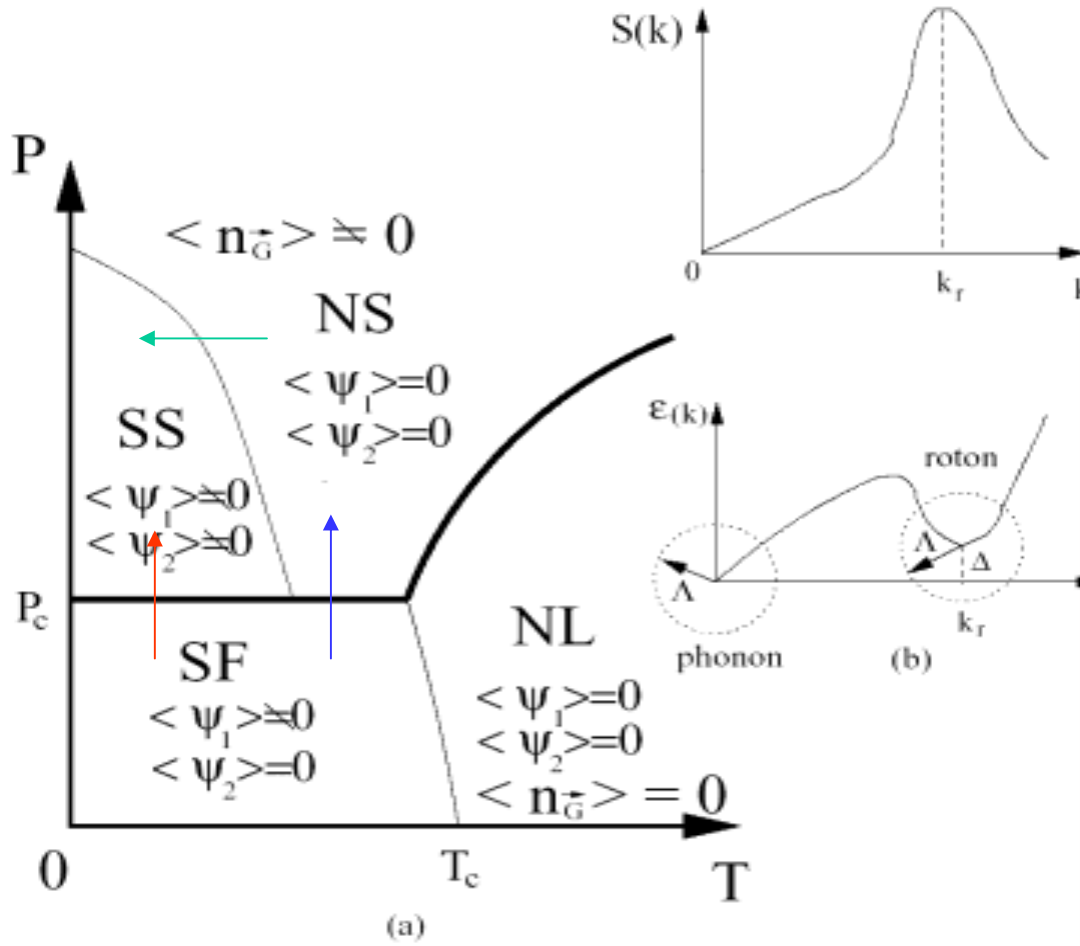
$$\psi_{ss} = \psi_0 \left(1 \pm \frac{2}{P} \sum_{m=1}^{P/2} \cos \vec{Q}_m \cdot \vec{x} \right), \quad \psi_0 = |\psi_0| e^{i\theta}$$

At mean field theory level:

The X-ray scattering from SS-v is the same as the NS-v, but will be modified by **Debye-Waller** factor

The X-ray scattering from SS-i has the **even-odd** modulation

Global phase diagram of He4 in case (2)



$$\omega_q = \frac{q^2}{2mS_q}$$

5. Excitations in a Supersolid

(1) **Superfluid phonons** in the ψ sector: θ

Topological **vortex** excitations in θ

(2) Due to the couplings, the **lattice phonons** \vec{u} in ψ sector are locked together with the n lattice phonons.

Only one kind of transnational symmetry breaking leads to one kind of lattice phonons \vec{u} .

A low energy effective action is *under* construction:

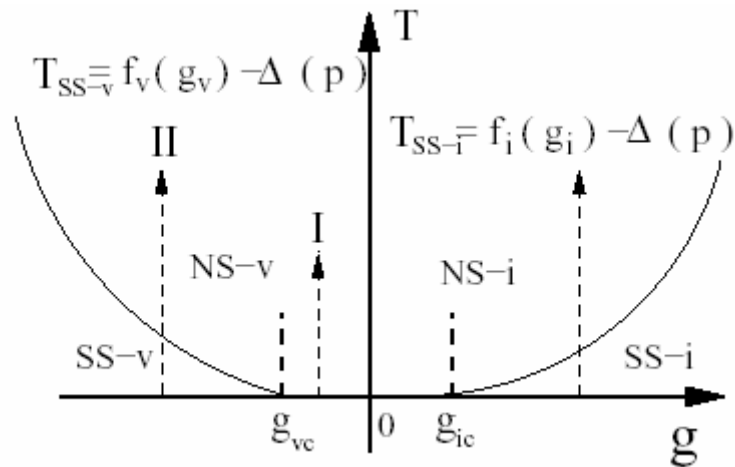
$$L_{\text{eff}}(\theta, \vec{u}) = L_1(\theta) + L_2(\vec{u}) + L_{\text{int}}(\theta, \vec{u})$$

If ignoring the coupling term $L_{\text{int}}(\theta, \vec{u})$

NS to SS transition is in 3D XY universality class:

$$f_{\psi_0} = K_{NS} |\nabla \psi_0|^2 + t_{NS} |\psi_0|^2 + u_{NS} |\psi_0|^4 + \dots$$

$$t_{NS} = T - T_{SS}$$



Very recent very refined Specific heat experiment at PSU detected:

Excess specific heat exhibits a peak $\sim 100 \text{ mK}$

The effects of the coupling term: $L_{\text{int}}(\theta, \vec{u})$

How phonon spectrum in a SS differ from that in NS ?

Debye-Waller factor: $I(\vec{G}) = e^{-\frac{1}{3}G^2 \langle u^2 \rangle} I_0(\vec{G})$

How the DW factors differ in the SS-v than the NS-v ?

How does it affect the X-ray scattering from the SS-v ?

Ongoing Experiments at PSU:

High precision X-ray scattering at 

6. Supersolids in other systems

Supersolid on Lattices can be realized on optical lattices !

*Jinwu, Ye, cond-mat/0503113, bipartite lattices;
cond-mat/0612009, frustrated lattices*

Cooper pair supersolid
in high temperature SC ?

The physics of lattice SS is *different* than that of He4 SS

Possible excitonic supersolid in electronic systems ?

Electron + hole \Rightarrow Exciton is a boson

There is also a roton minimum in electron-hole bilayer system

Excitonic supersolid in EHBS ?

There is a **magneto-roton** in **Bilayer quantum Hall systems**,
for very interesting physics similar to He4 in BLQH, see:

Jinwu, Ye and Longhua Jiang, cond-mat/0606639

Jinwu, Ye, PRL, 2006

7. Conclusions

1. A **simple** GL theory to study all the 4 phases from a **unified** picture
2. If a **SS** exists depends on the sign and strength of the coupling g
3. Construct explicitly the QGL of SF to NS transition
4. SF to SS is a **simultaneous** formation of SDW and normal lattice
5. Vacancy induced supersolid is certainly possible
6. NS to SS is a 3d XY with much **narrower** critical regime than NL to SF
7. Excitations in SS, phonon spectra, Debye-waller factors.....
8. X-ray scattering patterns from SS-v and SS-i

No matter if He4 has the SS phase, the SS has deep and wide scientific interests. It could be realized in other systems: